Leonardo and the Vitruvian Man

by Ernesto Fernández

(Extract from *"Platón, Leonardo y el sistema del Monte Carmelo"*)

Leonardo has created the most emblematic picture in Western iconography: the Vitruvian Man. Why is this figure so appealing? What has it meant for its creator and for humanity throughout the centuries?

Da Vinci belonged to the so called Victor Hugo´s Oceanic Men, who are defined by profoundness and scope to such an extent that the very notion of boundaries is out of reach

He did not discriminate Art, Philosophy and Science. His vision merged them into a unique process of knowledge:

"Painting is a philosophy", *"the most admirable body of science"*, *"the greatest mental reasoning"*, *"Nature´s self-begotten legitimate offspring"*. [1]

In the same fashion as Plato and modern science, Leonardo poses his design as a mathematical model of reality; he does not trace a mere man but a system of proportions, a metric structure which implies certain transformations.

According to Plato, knowledge applies to something that never changes, something that always remains invariable in the flow of phenomena: the Eidos, the Idea.

The acts of thinking and measuring are inextricably intertwined, the etymology of the term *think* is *weigh,* and the sanscrit root *ma* means *measure*, which in turn generates *mati*: reason, thought. The link between measure and think remains invariably in both acts.

Plato and his disciples (including Leonardo) have argued over these "performances" and the concepts deriving from them: symmetry and structure. Hence the metaphysical scope of the

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[1] Leonardo Da Vinci, Tratado de la pintura y del paisaje, Sombra y Luz, pag. 333/4,Editor Joaquín Gil, Buenos Aires, 1944 (excerpt translated by Patricio J. Cernadas)

polygons and polyhedrons and the inscription on the portal: "Nobody but a geometrician is allowed"

The fact of applying "the most perfect figure" to "the body of the world", and its quality of comprising the only five regular and available polyhedrons suggest a philosophy of Nature based on the logical-mathematical system of relations in which the principles of transformation and invariability are implicit. This is the core of the Platonic thought shared by Leonardo, and basically by modern Physics and Mathematics.

For Plato, the archetype of measure is inferred via geometrical exercise, a kind of "ordeal" concerning the methods of duplication of the square and the cube, which are known as the Delian Riddle (mentioned in some dialogues such as *Meno* and The *Laws)*.
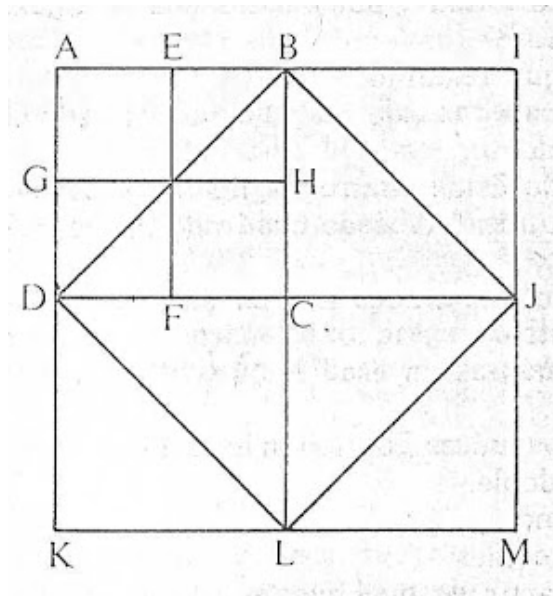

Meno - duplication of the square
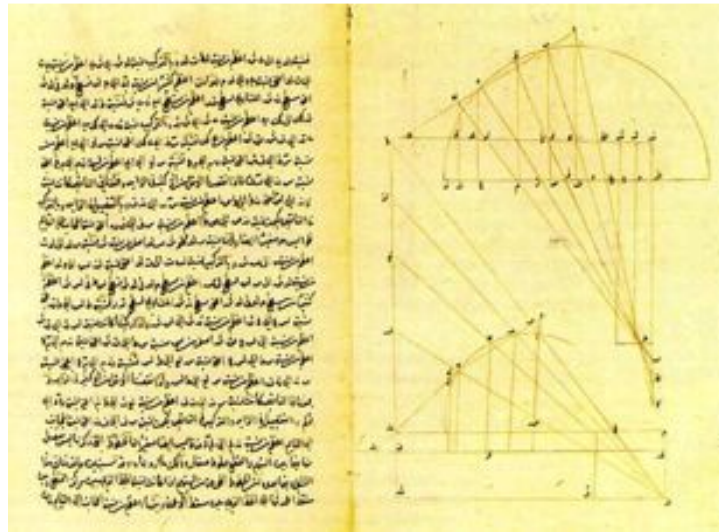
Duplication of the cube de Menecmo

The line, the square and the cube seem to be similar objects; the square is constructed with lines, the cube with squares, but when they are subjected to doubling it is evident that their generating principle is not the same. The pursuit of these transformations allows the transcendence of appearances. These methods of proportion and sympathetic metrics reveal the thread of the spirit, they turn out the geometrical proof of the world of the archetypes and the theory of the reminiscence.

The Vesica

Although not mentioned openly by Plato, several works seem to refer to a geometric structure made from two circles of equal radius which intercept in a way that their centers are caught within each other´s circumference. This scheme known as Vesica, understated among the successive vertex of all regular polygons is a geometric key to his own Metaphysics.

The Vesica stands for time and space metrics already mentioned in the Sulbasutras, writings older than Plato´s which describe how they are used for building temples and cities.
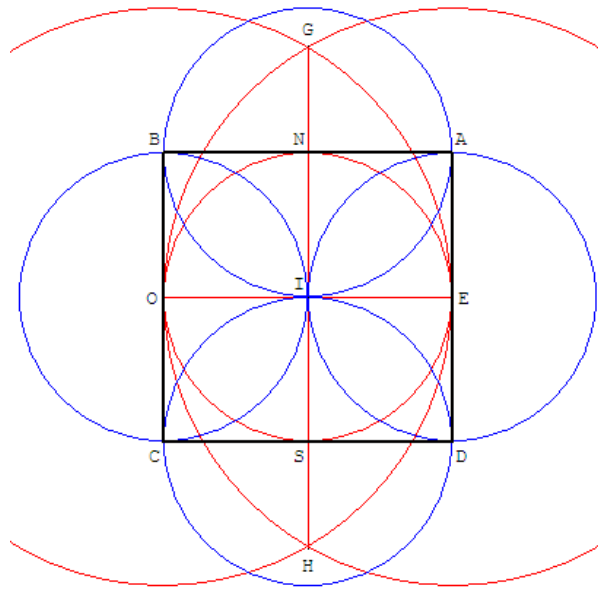
**Vesica -Sulbasutra, according to Marie-Noëlle Racine**

The most ancient mathematic formulation of the Vesica that has reached us was that of Euclid, a Plato´s disciple. This geometrician undertook the making of regular polygons with a ruler and a compass; a procedure that divides the circumference by a number of equal parts; the most elemental of the traced is the Vesica, which is analyzed in the First Proposition of the First Book of Elements: How to make an equilateral triangle on a given finite straight line.

LIBRO PRIMERO DE
LOS ELE MEN TOS
GEOMETRICOS DE EVCLIDES
philofopho Megarenfe.

Problema primero, propofition primera,

Sobre vna linea recta dada terminada hazer
vn triangulo equilatero.

Sea la linea recta dada terminada. A B. côuiene defcreuir
fobre A B. vn triágulo equi
latero. Sobre el cêtro. A. y
fegú elefpacio. A. B. defcri
bafe elcirculo. B. C. D. (por
la tercera petitiô) Y tambié
(por la mifma) fobre el cen-
tro. B. y eñl efpacio. B A. def
criuafe el otro circulo. A. C.
E. Y (por la primera peticiô)
defde el punĉto. C. donde los circulos fe cortan, tirenfe las li-
neas rectas, C A, C B. afta los punĉtos. A. B. Y porque el pun-
ĉto. A. es centro del circulo. C. B. D. fera ygual la linea. A. C.
a la linea. A. B. (por la decima quinta definitiô) Ité porque el
punto. B. es centro del circulo. C A E. fera ygual la linea. B C
a la linea. A B. luego ambas. C A. y la. C B. fon Yguales a la
linea. A. B. Y las cofas que a vna fon Yguales, étre fi fon ygua
les (por la primera comun fentencia) luego la linea. A C. es
ygual a la linea. C B. luego las tres lineas C A. A B. B C. fon
yguales entre fi. Sera pues equilatero el triangulo. A B C.
y fabricado fobre la linea recta dada terminada. A B. lo qual
conuino hazerfe.
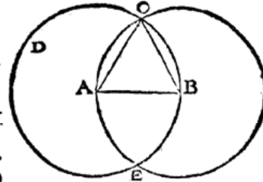
Euclid´s First Book of Elements and the Vesica

In various cultures the Vesica is the base of ritual proceedings of orientation joining the temple or city forms to the universe form. Vitruvian refers a similar ritual employed by Romans to establish the *cardo* and the *decumanus* in the foundation of their cities.

The mystery component of Platonism may have contributed to the concealment of the description of this structure as it is suggested in two statements from *Phaedrus:*

"In our condition of being initiated in those mysteries (…)" "(…) plenty, pure and quiet were the visions in which we have been initiated (…)"

I believe that the philosopher deliberately tried to conceal data that might allow laymen to identify them. In various passages form the Timaeus its figure seems to arise.

The three-part structure of the Vesica, which bears a resemblance to a Venn diagram, is somehow referred in the following enigmatic paragraph:

"God took of the unchangeable and indivisible and also of the divisible and corporeal, and out of the two he made a third nature, essence, which was in a mean between them, and partook of the same and the other, the intractable nature of the other being compressed into the same"[2].
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The Vesica has the quality of linking the square roots of the numbers 2, 3, 5 and 9 and the Golden Number:

$$\frac{1+\sqrt{5}}{2} = 1'6180339887\ldots$$
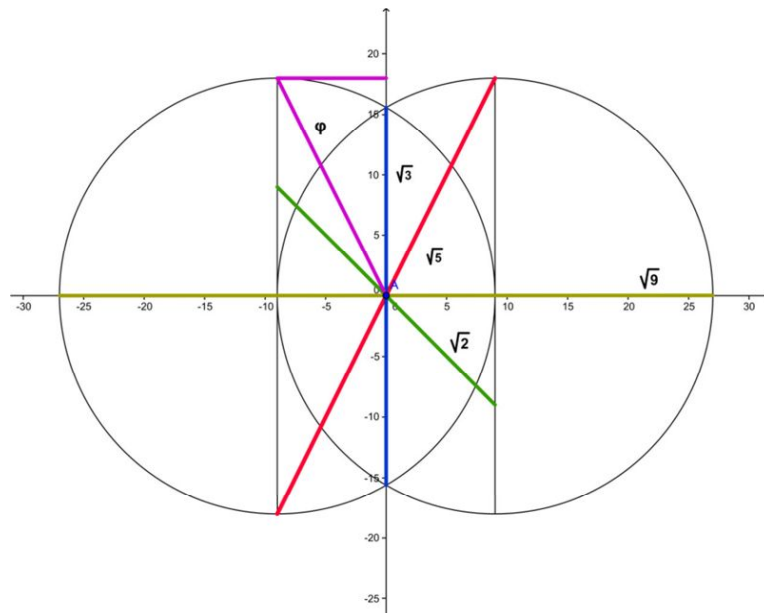
The Vesica and roots of 2, 3, 5, 9, and the aureum number

Another text describing the procedure of decussation, on account of its resemblance to the Latin word X:

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[2] PLATO, *Timaeus* , Section 1, The Gutemberg Project, 2008.

"The entire compound was divided by him lengthways into two parts, which he united at the center like the letter X, and bent into an inner and outer circle or sphere, cutting one another again at a point over against the point at which they cross."[3].
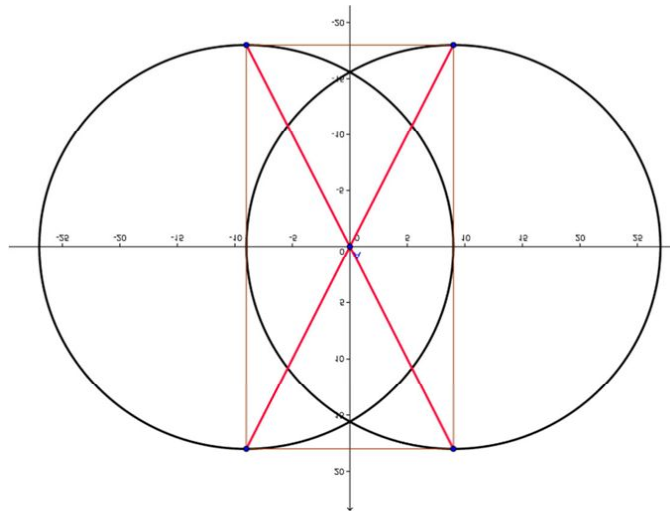
Vesica and C*rux decussata*

Both quotations from the Timaeus are analyzed by Carl Gustav Jung and connected with the Christian Trinitarian conception and the layout of a city with crossed streets, motifs associated with the Vesica. As for the upon the first of them, he observed that

"For the Egyptians the x in a circle meant the soul of the world, according to Porfirius. In fact this is the hieroglyph related to "city", therefore I suspect that Plato tried to unveil the Mandala structure which later on arouse in *Critias* as the capital of Atlantis."[4]

Both the Vesica and the remaining geometric elements of the Platonic conception, the regular polygons and polyhedrons and

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[3] Plato, *Timaeus* , Section 1, The Gutemberg Project, 2008.

[4] C.G.Jung Simbología del espíritu, pag 244,Fondo de Cultura Económica 1962 México

the motifs of doubling the square and the cube are touchstones in Leonardo´s conception.

The diagrams in which he studies geometric transformations often present multiplications or divisions of the square by two. In another writing on the perspective to the Atlantic Codex he observes that "as the space is duplicated, the size reduction is also duplicated."
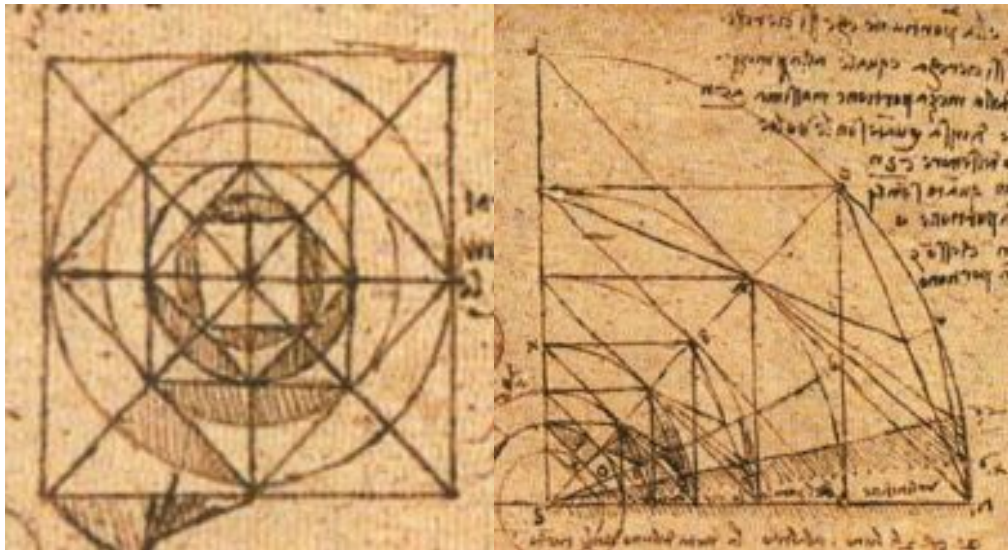

Duplication of the square– Atlantic Codex

The Delian problema of the duplication of the altar is also analyzed by Leonardo on many occasions.
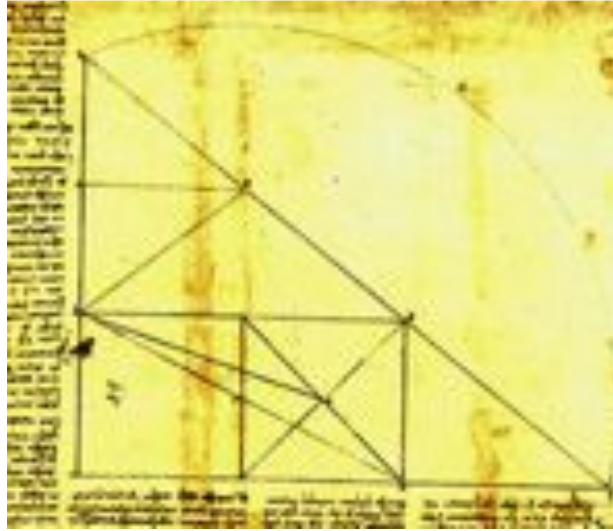
The Delian Problem– Atlantic Codex

The Vesica is already present in the works of Andrea del Verrocchio and appears a lot in the designs of Leonardo; under sea waters, machines, organisms and emblems it is revealed as the metric core of his thought.
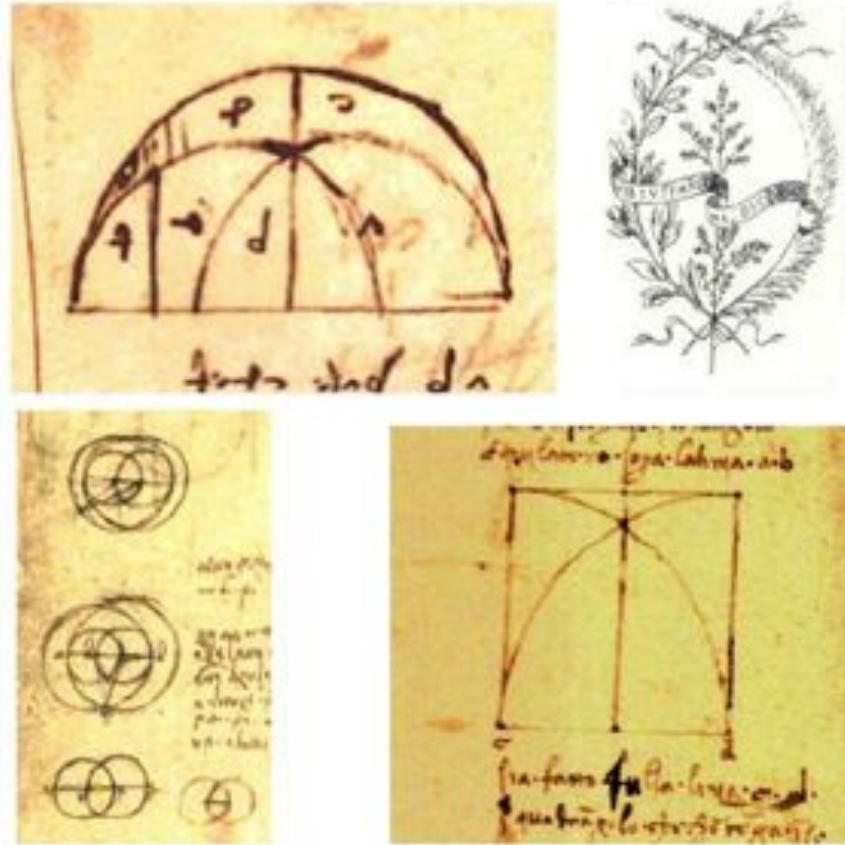
Vesicas by Leonardo da Vinci

The conception of Da Vinci is Platonic and his well-known design represents not only the finest illustration of the Greek geometric Metaphysics, but also a self-portrait. It shows Leonardo thinking himself over. A circle and a square frame him and his thought reminisces the Socratic maxim "Know Thyself", whose maieutic tools are the ruler and the beam compass. How did he design this renaissance mandala? What are the proportions of both figures?

The Vitruvius Treatise, "one of the darkest and creepiest books in Latin literature" according to Menéndez y Pelayo, is read by the Renaissance scholars without its original illustrations. Leonardo´s drawing is the genial interpretation of a mystifying text. Most of the illustrators, Cesariano and Martini among them, developed designs in which the circle and the square are displayed

concentrically. Leonardo´s view introduces asymmetry, two figures overlapping on a common central axis. The *homo ad circulum* is on the belly and the *homo ad cuadratum* is above the genitals. The chartered man displays additional legs and arms; however, the beauty of the design resists asymmetry.

This apparent irregularity may have puzzled the researchers. Symmetry seems to refer to a hidden plan, an animated geometry The multiple limbs trace a jump, typical of a "geometry created upon movement" devised by the author.

Analysts do not mention the ratio between the diameter of the circle of *homo ad circulum* and the side of the square containing the *homo ad cuadratum*, which is 1.20. Like the "discarded stone", this proportion enables the observation of "the temple of the body".

Leonardo does not make it explicit, but nevertheless frames the Golden Man in a square of 900 units[5], in the fashion of other illustrators who also considered Vitruvious´ calculations: the length of the outspread arms is equal to the height of a man; from the hairline to the bottom of the chin is one-tenth of the height of a man**.**

Leonardo virtually[6] includes the extension of the 900 square, which is 1296[7], determined by the circle based on the first square. The ratio between the sides of both squares is the already mentioned 1.20.

The square to his system which contains the Golden Man is built using the vesica and other internal elements shown below.

[5] 30² square as found in Jean Martín y Cesare Cesariano.

[6] Figure not drawn by Leonardo.

[7] 36² square

Fig. 1　　　　　Fig. 2
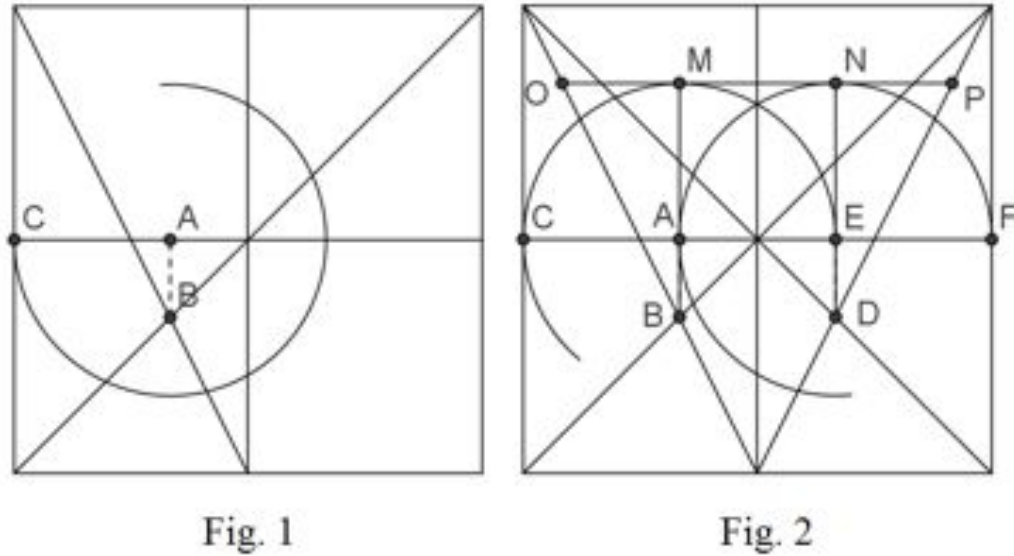
After tracing the diagonals he obtains the point B from Figure 1 and the center A after a vertical line goes through the circle whose radio is AC. The same operation is repeated on the right side of Fig. 2 for center E of the circle whose radio is EF. The points M and N are determined upon the intersection of vertical lines BM and DN with the circles. The segment OP derives from the intersection of the imaginary straight line running along M and N upon its own intersection the sides of the diagonal that links the middle point of the base of the square and the its border .

The verticals running down O and P build the internal square in figure 3 where the Golden Man is placed. The relation between both squares is 1.44. The Vesica turns out the hidden skeleton of the Vitruvian Man.
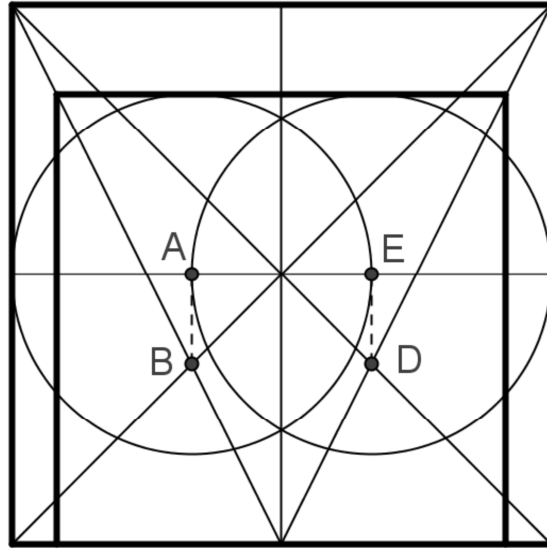
Complete construction

Contrary to what prestigious researchers have stated about him, Leonardo does not withdraw from the medieval tradition rediscovering the metric archetype of Vitruvius. Through the same structure he finds that Hildegard has discovered what Mystics did some centuries before, which is similar to the oldest Romanic tapestry ever preserved: "The Creation".
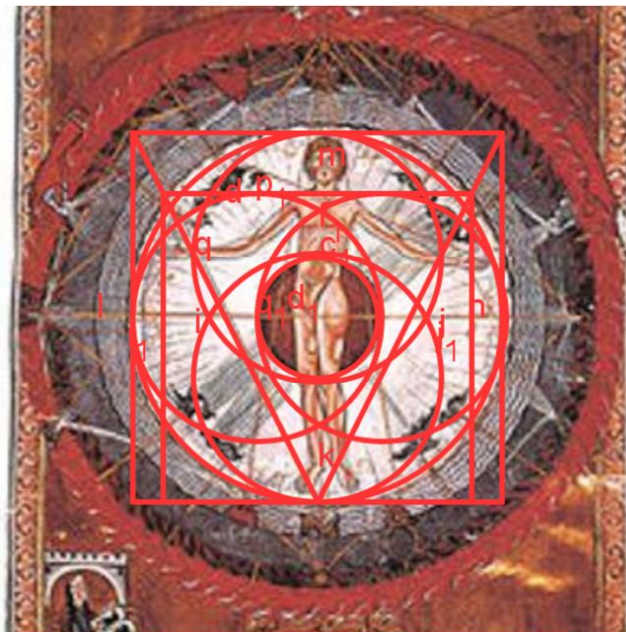
Hildegarda – Microcosm, Golden Man

Tapestry of Creation, XII Century-Gerona Cathedral

After devoting several years to theological writings, Pascal grew a interest in cycloid, epicycloid and hipocycloid curve planes in 1657.

It is possible that figures such as the pentalpha, which was admired by Pitagoric, or some other type related to curve planes, might have been used to represent image of the Universe as created by God.

It was nicknamed as "Helen of Troy" because it had seduced mathematicians and philosophers. The cycloid was studied by Nicholas de Cusa, Mersenne, Galileo and Descartes, among others

Pascal made a four-part gear wheel mechanism to analyze its properties and praised them poetically:

"(...) they are not just spring flowers depending on the seasons, but some which have been selected from the finest geometry gardens; in fact, they turn out amaranths that will never wither." (Blas Pascal; a letter to Sluse.)

The hypocycloid is the curve stated by a point in a circumference which revolves within another circumference without skidding. Due to its dimensions, Leonardo´s design encompasses they

hypocycloid generated by the fore mentioned pentalpha, which is a polygon related to the Golden Number. The circle of the Vesica rotates and generates a nonagon. Perhaps the study of its mechanisms laid in the foundation of this development whose mathematical formulation would be tackled by Pascal himself.
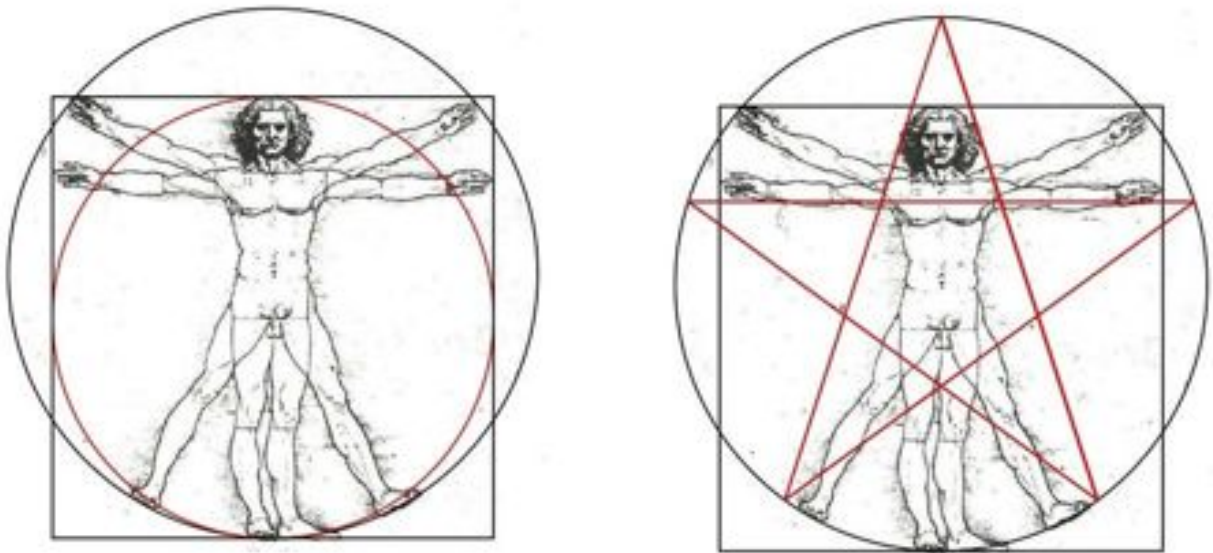
Pentalpha e Hipocycloid within en Leonardo´s Man
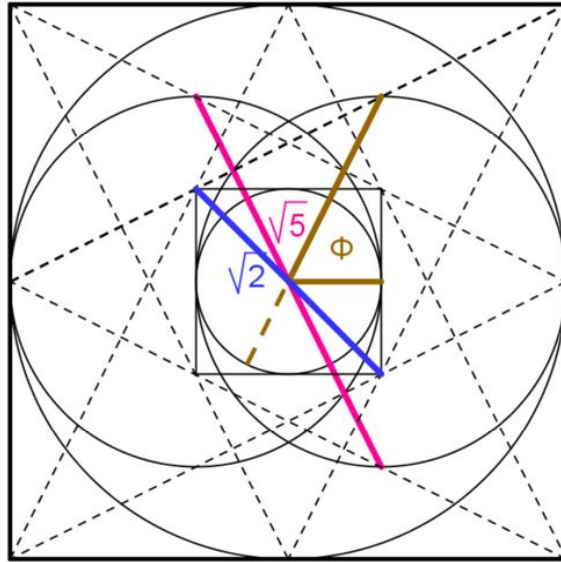
Tons Brunés Star

Leonardo traces various segments on the body of his model (highlighted in red in our figure) which indicate the exact division of the square in which it is placed by means of an octogonal star; this figure is made with eight diagonals joining the common points on the sides of the square and the corner points, the side of the star is $(\sqrt{5})/2$ of the side of the square. This division is known as the star of Tons Brunés, a Danish architect who employed it on the analysis of classic buildings and paintings.
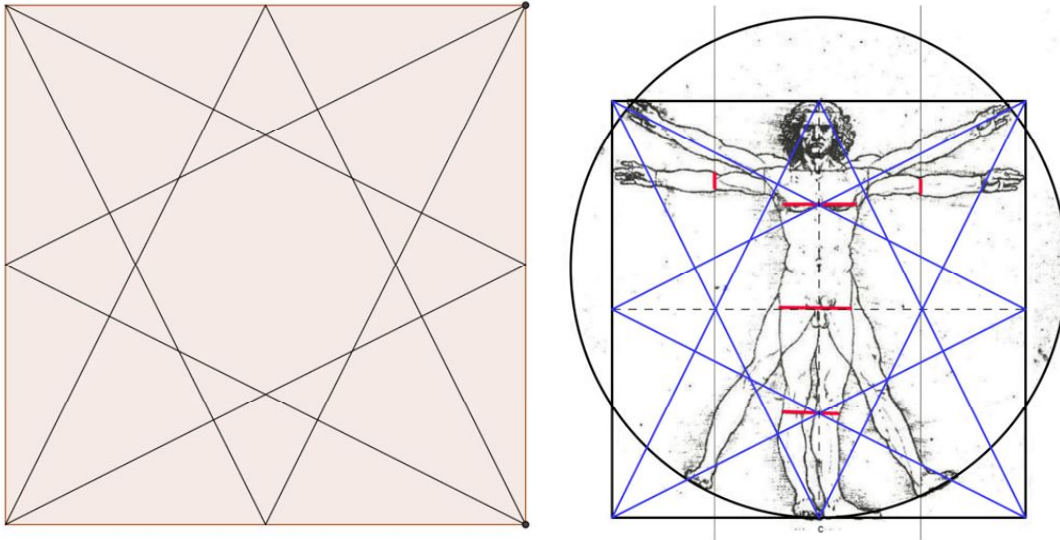
Star of Tons Brunés y Leonardo´s Man

Brune´s analysis, which he himself called "sacred cut", is related to the duplication and triplication systems mentioned before. This does not take into account the common relations shared by the Star, the Vesica and the Golden Man as seen in the drawing: the division by 3, √2, √5 and ø. [8]

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[8]The division by 3 es related to the side of the central square with respect to the side of the bigger square within which it is contained.

**Vesica and Brune´s Star - division by 3, √2, √5 and ø**

Another way of finding the Golden Number is toppling the side of the star, as it is described below
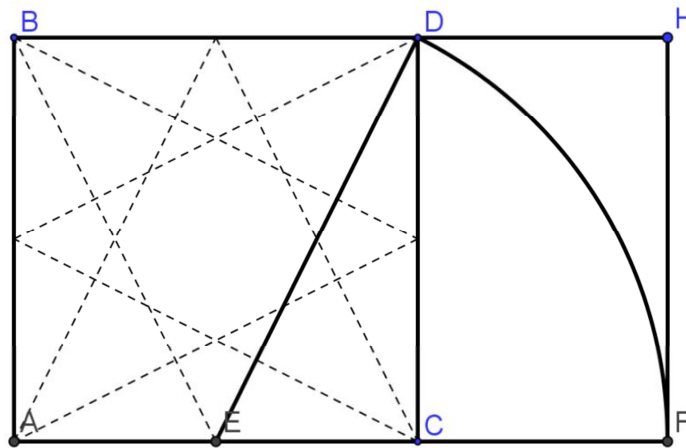

As the square ABCD, whose side AC = 1, and the side of the Star ED = EF determine the quotient ED/AC = (√5)/2 = 1,11803398…., which once toppled generates the golden rectangle ABHF. Then (ED/AC) + EC = CF + AC = side of the golden rectangle la[9] = EF = 1,618033989... (Golden number)[10].

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[9] Euclyd develops the construction of the golden rectangle form a square in his proposition 2.11.

The metric archetype of Leonard is recognizable in various works of Western Art. Here are some examples.
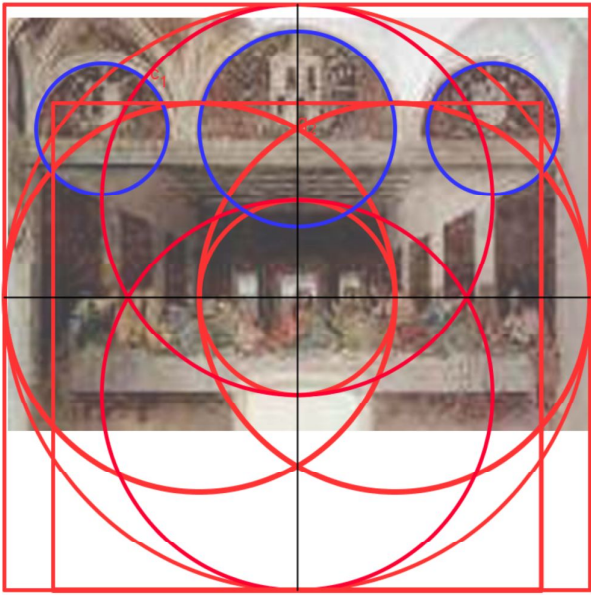
The Last Supper
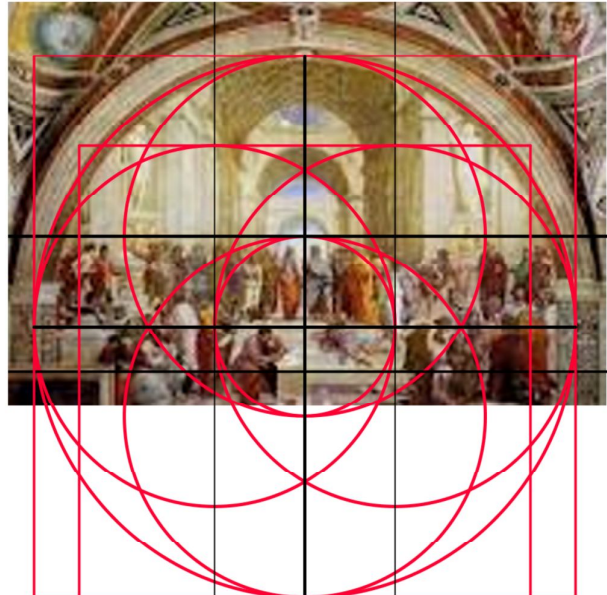
The School of Athens

Leonardo employs the metric archetype analyzed as the base of mural making. The perspective of the painting shows the big square and the internal square whose horizontal side lies on the Vesica. The center of the painting is Jesuschrist´s head, which is the fugue point of the perspective. The structure of the Vesica employed also determines two axis, one horizontal and the other vertical, which split the surface in two equal sections. The horizontal runs through the heads of all disciples except Judah.

Rafael de Sanzio employed Leonardo´s measurements in "The School of Athens". The first portal corresponds to the circle within the big square, whereas the second is set within the circle of the Vesica. The remaining portals correspond in height to $\sqrt{3}$ and the central circle is 1/9 to the big circle.

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[10] It can be proved that the addition of the sides of the star, plus the perimeter of the square in which it is contained, is equivalent to 8 times the golden number of the side of the square.

The horizontal axis running through the center of the principal circle is in fact the center of the work, from which the work itself is projected.

Albrecht Dürer analyzed the works of philosophers and artists (among them Varro, Equicola, Barbari, Alberti, Pacioli and especially Leonardo) focusing on the metric pattern of the Man deriving from Plato. The ideal measurement and the action of measuring are Dürer´s bone of contention; the artist is the one who definitively can measure, hence the title of his treatise "Instructions for measurement".

Dürer interprets and comprehends Leonardo´s metrics as a system of continuous transformations in which some qualities are invariable. This identification can be verified via the superposition of his own design of the ideal human proportions with Leonardo´s.
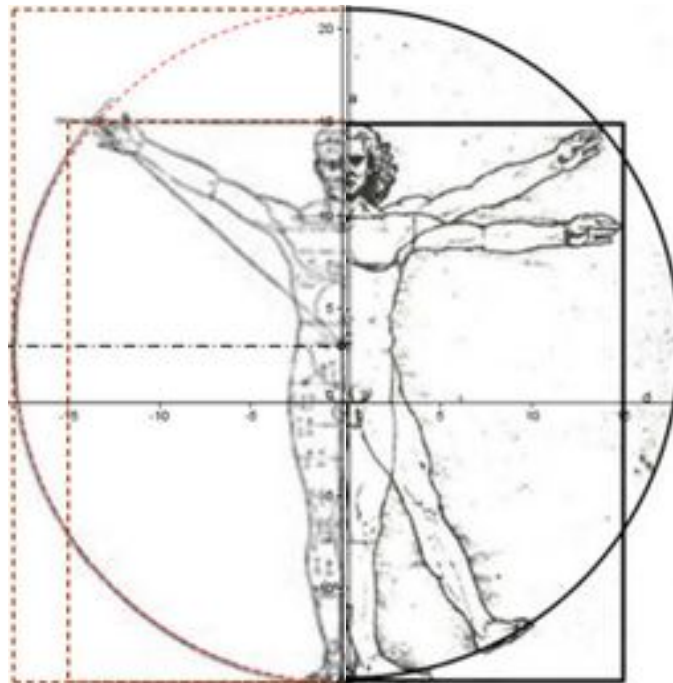

Golden Man– Leonardo y Dürer´s superposition

Two centuries later, Gauss delved into the theory of numbers and pondered on the divisibility properties of the perfect squares:

"The principles related to the division of the circle in seventeen parts…"

Thus Gauss disproved the supposition stating that the only constructible regular polygons were those included in Euclyd´s "Elements" (a deeply rooted supposition held for twenty centuries).

Johannes Erchinger provided a method for constructing Gauss´ heptadecagon from which we show three of the sixty-four steps achieved, the vesicas necessary for the construction can be observed.
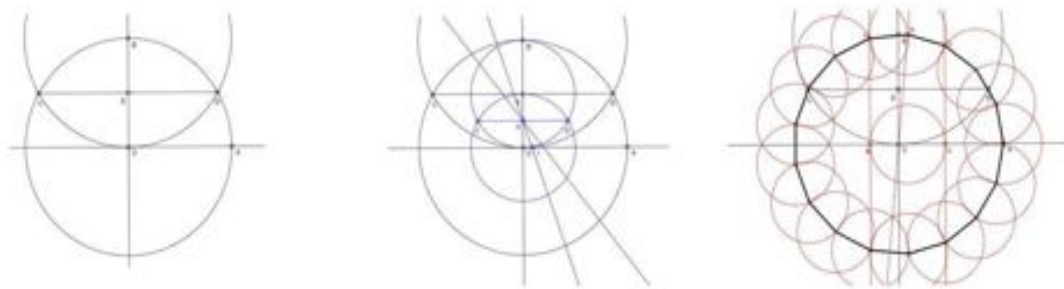

Johannes Erchinger Method for constructing Gauss´ heptadecagon

The Vesica does not appear casually in the construction of the heptadecagon. In fact, it lays in the core of the mathematician´s speculation. Gauss makes clear that his discovery "is not really more than a corollary to a "major theory" yet not fully developed." The mainstream body of this "major theory" is "Artithmetic Disquisitions", a treatise in which he develops the "Modular Arithmetic", a system which allows solving certain problems of whole numbers. When the well-known test of nine is done, another modular arithmetic operation applies, one in which the divisor is 9.

"Arithmetic Disquisitions"continues the metric conception which Pitagor and Plato passed onto Nicolás de Cusa and Leonardo da Vinci.

The modular arithmetic has been nicknamed as "clockwise arithmetic" because the numbers turn when they reach the value denominated "Module". The Platonic Vesica appears in the Euclydean or whole division, which is in the foundation of the modular arithmetic.
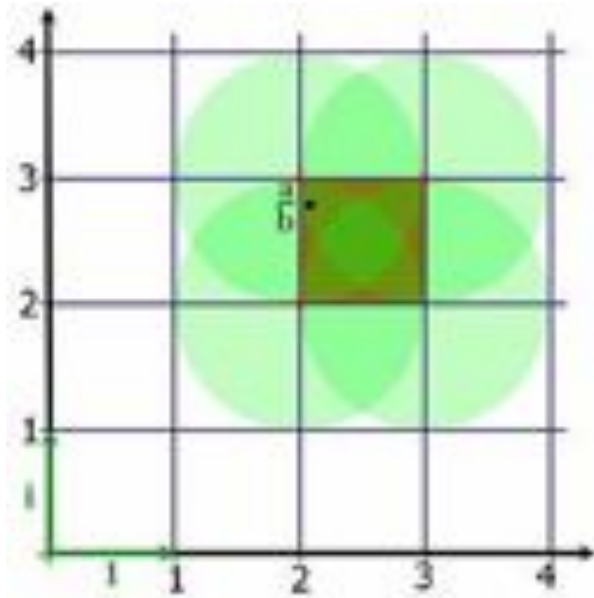
Vesica – Ilustration of the Euclydean division among the Gaussian whole numbers

Da Vinci´s world view includes two seminal mighty branches of modern mathematics: Topology and Group Theory.
According to Fritjof Capra in "Leonardo´s Science":

"When we contemplate Leonardo´s geometry from current mathematics (…) we can acknowledge that it has developed the beginnings of the Math's branch which today is known as Topology. Leonard´s Topology is also Geometry of continuous transformations o correspondences. Some general properties equivalent from the topological viewpoint are preserved."[11]

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[11] Fritjof Capra, La ciencia de Leonardo, pag. 270, Ed. Anagrama Barcelona 2011.

Intuitively the Florentine employed the current "Leonardo´s group" in order to add chapels and niches to the central core of a church without altering it symmetry.
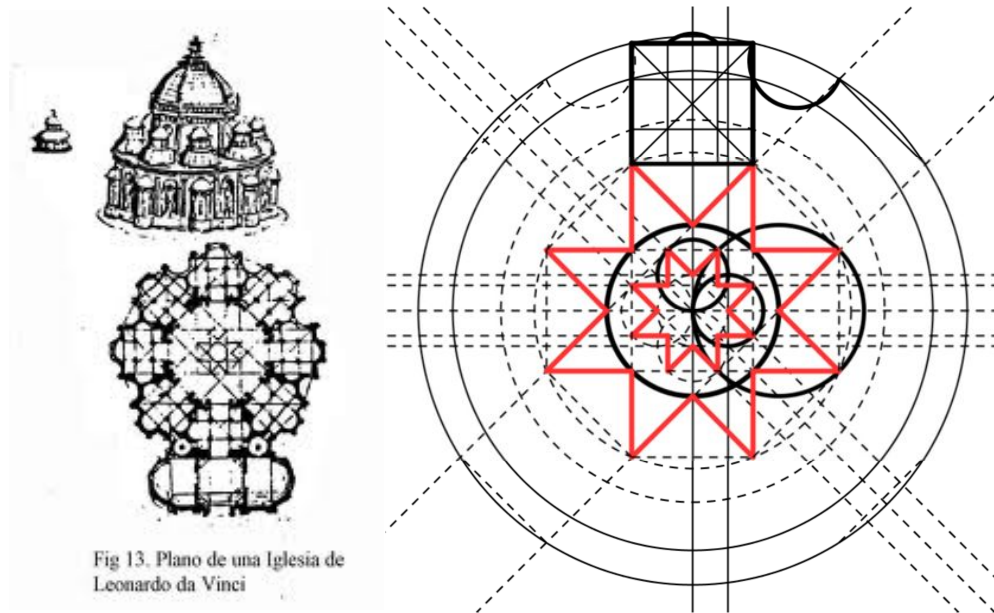

Fig 13. Plano de una Iglesia de Leonardo da Vinci

Church designed with a group by Leonardo - Analysis

The Theory of Algebraic Groups has a fundamental kinship with the notion of symmetry and represents the mathematic formulation of its own properties. We can define a symmetry as something that remains invariable before the action of a group of transformations.

The application of these mathematical structures in areas so diverse as Crystallography, Microphysics and Anthropology has demonstrated the profound relationship between symmetry and the structure of Reality. In Physics of XXth Century[12], Víctor F. Weisskopf has asserted that:

"(…) the symmetry of a flower is somehow determined by the fundamental symmetry of a quantum atomic state".

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[12] Weisskopf, Victor F, La física en el siglo XX, pag. 220,Alianza Universal, 1972, Madrid

In "Mathematical Physics" the theorem of Emmy Noether demonstrates that all magnitude "maintained" in a physical system is the result of symmetry; the maintenance of energy and amount of movement in physics and the maintenance of the probability in the transformation of the statistical laws are examples of this invariance.

Einstein´s theory of relativity represents a geometric model producing symmetrical equations known as Lorentz transformations, which trace space and time rotations.

In his master lecture on philosophy and group theory, the philosopher and mathematician Cassius J. Keyser states that the utmost stimulus of man is replying the question: What is it that remains? The quest for invariance was the fundamental motive of Art, Religion and Science.

Keyser poses the key question: Is the mind an Algebraic Group? Although he is hesitant about recognizing this quality in the mental structure of an ordinary man, he thinks it is possible in extraordinary cases:

"A genius belonging to the so called universal type, such as Aristotle, Leibnitz or Da Vinci, may possess a mind which has the specific quality of that group."[13]

The mathematician mentions that a similar idea has come up in the minds of numerous thinkers since the most remote ancient times: the traditional notion of the cosmic cycle or year. After analyzing the closed system of transformations in the universe as presented in "De Rerum Natura" by Lucretius he puts forward Scipio´s Dream by Cicero and its structure of nine isomorphic circles instead of the other traditional systems by Dante, Hildegard and Bahá´í on Mount Carmel.

Drawing on Whitehead and Eddington, the epistemologist and mathematician Luciano Allende Lezama has developed a system called Toposophy in which he formulates a conception of thought and language and its links with the physical universe by means of

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[13] Newman, James R., Sigma, El mundo de las matemáticas, Tomo 4 pag 347, Ed Grijalbo, 1983, España

the logical interrelation of circles and the Algebraic Group Theory.

Lezama´s platonism is evident:

"The physical world is ongoing, changeable; however, there is something invariable at the bottom of all beings and things. These permanent entities are those sought by Plato (...)[14]

All the reasons that we have identified in Leonardo form his theory. The duplication of the square is present in both his "conceptual structure of the atom" and his "configuration field".
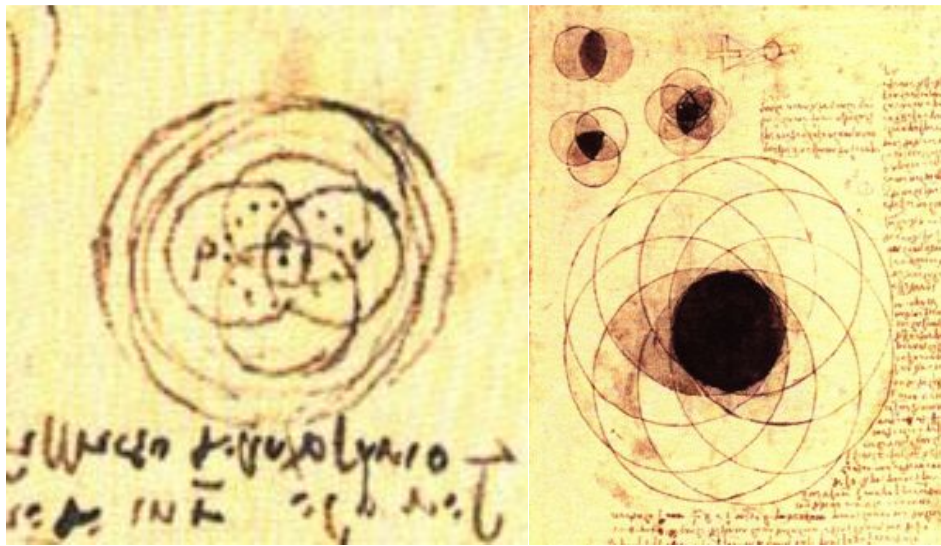
Secant Circles– Leonardo, Atlantic Code

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[14].Allende Lezama, L.P.; Hombre, Mundo, Trascendencia; pag.182, Ed. Asociación argentina de Epistemología, 1964, Argentina
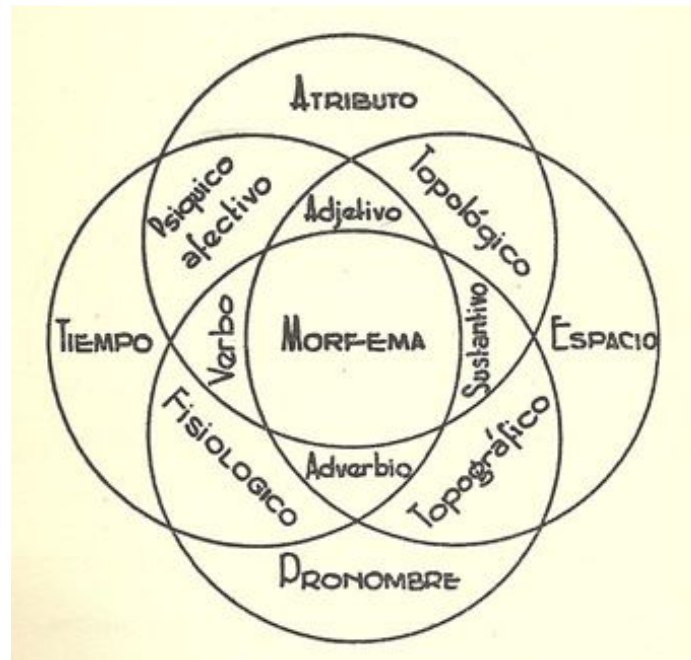
Secant Circles by Allende Lezama

The group theory is in the core of Toposophy:

"He have observed the concept of "invariance" as one of the most elementary in all rational speculation and we have applied it in the same guise as mathematicians since Klein formulated his "Erlangen Program" in which he restricted geometry to the study of a single group (...)"[15].

He also claims that the golden number and the pentagon are of singular importance, the golden ratio stands for a fundamental invariable of the topological thought and structure of the physical world.

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[15] L.P. Allende Lezama, Hombre, Mundo, Trascendencia; pag. 83, Ed Asociación argentina de Epistemología 1964 Argentina
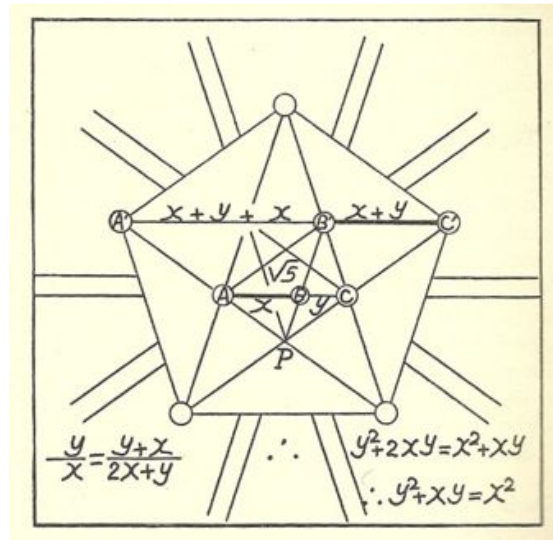
A. Lezama –Euclidean and projected representation of the golden section

Lezama, whose conception is similar to Keyser, defines Topology as "The very structure of the human intellect"

The Vesica is in the core of Allende Lezama´s system. The epistemologist relates it to both the human mental structure and Lorentz´ transformation, which is in the core of contemporary Physics:

"Lorentz transformation, which is a double trigonometric ratio, requires a configuration of secant circumferences of a common radio unit. The secant circles in which a circumference passes through another one´s center determine the meridians that we call Lorentzian. These meet the requirements of the double trigonometric ratio of Lorentz´ transformation."[16]

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[16] Allende Lezama, Luciano Pedro: Hombre Mundo Trascendencia, pág. 152, Edición de la Asociación Argentina de Epistemología, 1964, Buenos Aires.
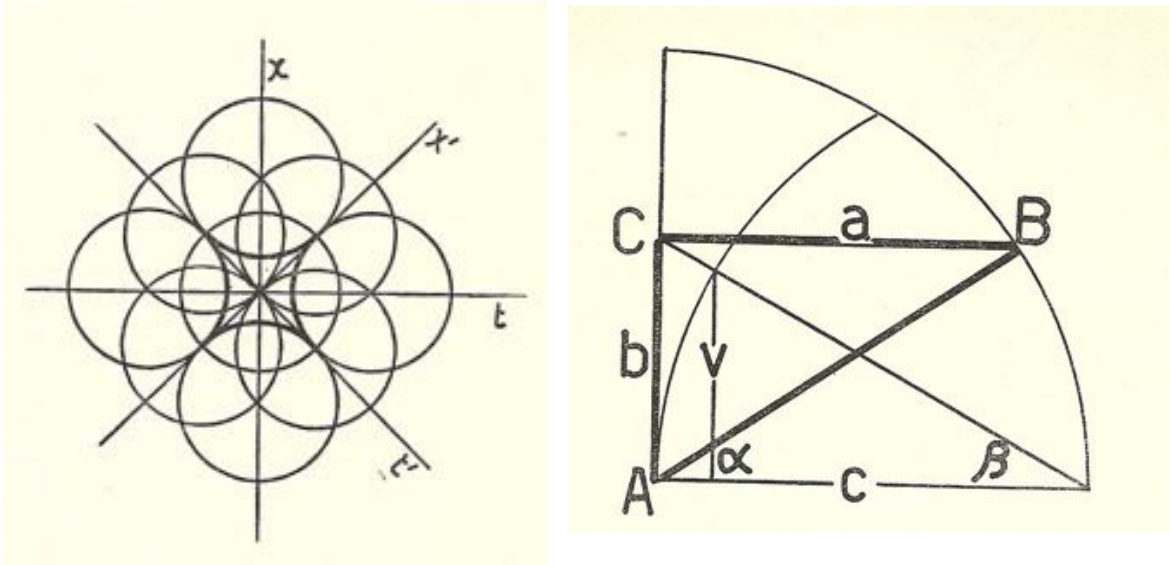
Allende Lezama – Lorenztian Meridians and Lorentz transformation

Conclusion

The scope of this review from Plato to modern thinkers enables a provisional answer to the first question of this essay:

Leonardo´s man represents the invariable essence of human Nature; his nudity is an emblem of Adam, the first man that all men incarnate in their singularity; his double crucified figure relates to Christ, the second Adam, symbol of pain and plenitude. It illustrates a scientific, philosophical and artistic model that guards the ecstasy of the wholeness for us, because we are accustomed to the segmentary and incomplete. As it was revealed in the Far East: "The man is the supreme talisman".